Type II Calabi-Yau compactifications in general spacetime signature

Maxime Médevielle (University of Liverpool)

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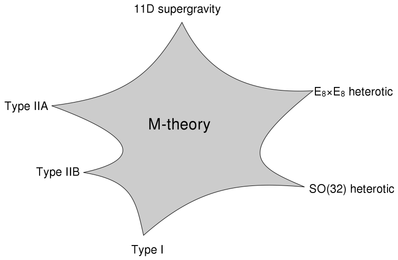
with Thomas Mohaupt and Giacomo Pope

Spacelike T-duality

String theory is a web of perturbatively defined theories related to each other by various dualities.



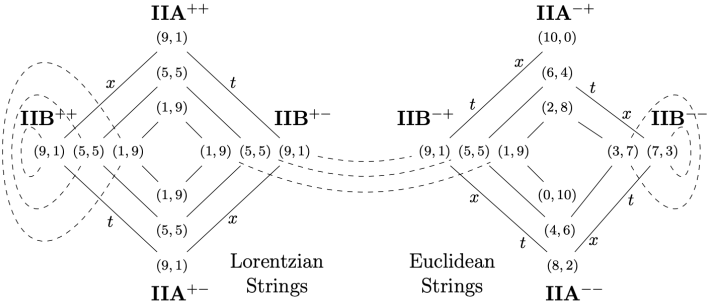
Consider the circle compactification $\mathcal{R}^{1,8} \times \mathcal{S}^1$: Type IIA on radius R and Type IIB on radius 1/R are equivalent.

Timelike T-duality

Doing a timelike T-duality gives new theories : IIA* and IIB*.

[Hull '98]

With S-duality we uncover a web of theories realising all maximal susy algebra and spacetime signatures.



[diagram from Dijkgraaf et al.]

Supergravity

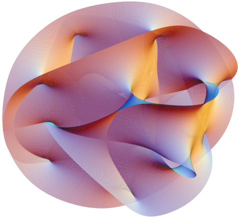
- Type IIA theories are :

$$S = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{|g|} \Big(e^{-2\Phi} \left[\mathcal{R} - \frac{\alpha}{2} |H_3|^2 + 4(\nabla\Phi)^2 \right] - \frac{\alpha\beta}{2} |F_2|^2 - \frac{\beta}{2} \left| \tilde{F}_4 \right|^2 \Big)$$

and Type IIB theories :

$$S = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{|g|} \Big(e^{-2\Phi} \left[\mathcal{R} - \frac{\alpha}{2} |H_3|^2 + 4(\nabla\Phi)^2 \right] - \frac{1}{2}\alpha\beta |F_1|^2 - \frac{1}{2}\beta \left| \tilde{F}_3 \right|^2 - \frac{\alpha\beta}{4} \left| \tilde{F}_5 \right|^2 \Big)$$

Going to 4D



Calabi-Yau compactifications of Type II preserves some Susy, allowing to have exact non-perturbative results while keeping a rich dynamic.

After compactification we obtain $\mathcal{N} = 2, D = 4$ Supergravity theories coupled to

Vector multiplets	A_μ, z	1 gauge field, 1 complex scalar
Hypermultiplets	q	4 real scalars

Special Kahler geometry

We obtain the following Lagrangian

$$L_{G+VM} = \frac{1}{2}\star R_4 - g_{\alpha\bar{\beta}}(z,\bar{z})dz^\alpha \wedge \star d\bar{z}^{\bar{\beta}} - \frac{\lambda}{4}\mathcal{I}_{IJ}F^I \wedge \star F^J + \frac{1}{4}\mathcal{R}_{IJ}F^I \wedge F^J$$

for Vector multiplets, the geometry is special (para-)Kahler, which means that there exists a holomorphic function $F(z)$ that determines $g_{\alpha\bar{\beta}}(z,\bar{z})$, $\mathcal{I}_{IJ}$ and $\mathcal{R}_{IJ}$.

Hermitian geometry	$i^2 = -1$	$\bar{i} = -i$
Para-hermitian geometry	$e^2 = 1$	$\bar{e} = -e$

Quaternionic Kahler geometry

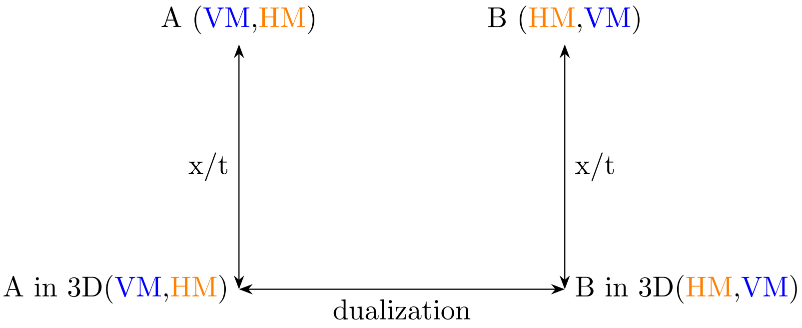
For Hypermultiplets, the lagrangian is

$$
\begin{aligned}
L_{HM} =& - h_{MN}(Q)\partial_\mu Q^M \partial^\mu Q^N \\
\underbrace{=}_{CY3} & - G_{\gamma\bar{\delta}}(q,\bar{q})dq^\gamma \wedge \star d\bar{q}^{\bar{\delta}} - \frac{1}{4}d\phi \wedge \star d\phi \\
& + \epsilon_1 e^{-2\phi}\left[d\tilde{\phi} + \frac{1}{2}\left(\zeta^I d\tilde{\zeta}_I - \tilde{\zeta}_I d\zeta^I\right)\right]^2 \\
& - \frac{\epsilon_2}{2}e^{-\phi}\left[\mathcal{I}_{IJ}d\zeta^I \wedge \star d\zeta^J - \epsilon_1 \mathcal{I}^{IJ}\left(d\tilde{\zeta}_I + \mathcal{R}_{IK}d\zeta^K\right)^2\right]
\end{aligned}
$$

where X^2 really means $X \wedge \star X$.
The geometry is (para-) quaternionic Kahler, characterized by their holonomy group $\left[Sp(n) \times Sp(1)\right]/\mathbb{Z}_2$.

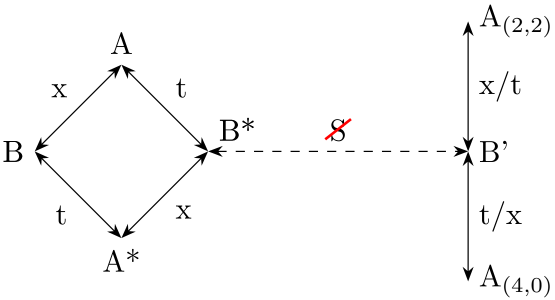
T-duality in 4D

Just like in 10D we can relate theories by a space/time-like circle dimensional reduction, realizing a T-duality.



4D duality web

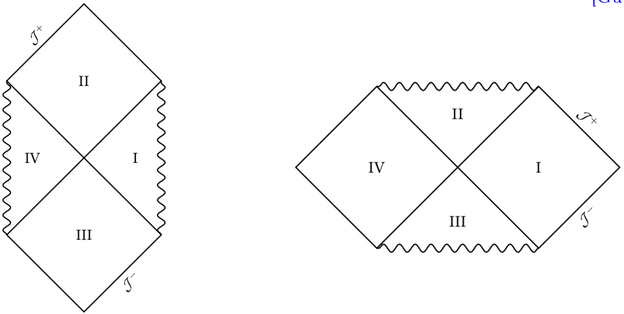
- We can map out the complete duality web of theories in 4D — pure spacelike, pure timelike and mixed T-dualities.
- The 4D duality web is a projection of the 10D one, however we have generically two orbits : One Lorentzian and one with signature change.



BH/Cosmological solutions

In previous work some solutions of these theories were found describing cosmological and black hole solutions, which share the same thermodynamic properties.

[Gutowski et al.]



Conclusion

- String theory provides a framework to study dimensionality and signature of spacetime.
- This gives rise to interesting theories in 4D — we gave a full description of the duality web relating their scalar geometries.
- Solutions of exotic 4D-theories can now be studied from a microscopic point of view (uplift in terms of D/E-branes).

Thank you :)

Type-II Calabi-Yau compactifications, T-duality and special geometry in general spacetime signature

M. Medevielle, T. Mohaupt, G. Pope

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Extra slide

- Signature change can only be addressed in a quantum theory of gravity.
- Signature (0,4) : instantons, solitons, Hartle-Hawking, complex metrics as saddle points of the EQG path integral, ...
- Signature (2,2) : amplitudes/BH in Klein space, Topological String theory, Magical String, Twistors,...
- Exotic theories related to negative branes, non unitarity, negative energy states, super gauge groups, etc...
- Looking at exotic vacua might teach us about some fundamental properties of the theory (analogy with Higgs in the Standard Model)